GENERAL

analytic geometry i problems and solutions

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Analytic Geometry I: Problems and Solutions

Analytic geometry, also known as coordinate geometry, is the bridge between algebra and geometry. By assigning algebraic coordinates to points in a plane, we can describe geometric figures with equations and solve problems that would otherwise be tedious or impossible to tackle using pure Euclidean methods. In this comprehensive guide, we focus on the foundational topics of Analytic Geometry I, presenting a collection of carefully chosen problems along with step by step solutions. Whether you are a high school student preparing for exams, a college freshman taking your first course in mathematics, or a lifelong learner looking to sharpen your analytical skills, this article will provide you with clear explanations, illustrative examples, and a wealth of practice problems.

Why Study Analytic Geometry?

Analytic geometry equips you with a powerful toolkit for:

- Representing lines, circles, parabolas, ellipses, and hyperbolas using algebraic equations.
- Computing distances, midpoints, slopes, and intersection points efficiently.
- Translating geometric problems into algebraic form for easier manipulation.
- Building a foundation for advanced topics such as vector calculus, differential geometry, and computer graphics.

Mastering the basics of Analytic Geometry I sets the stage for deeper exploration into higher dimensional geometry, complex numbers, and even physics.

Key Concepts Covered in Analytic Geometry I

Coordinate System and the Cartesian Plane

The Cartesian plane consists of two perpendicular number lines: the horizontal x axis and the vertical y axis. Every point in the plane is uniquely identified by an ordered pair (x, y). The distance from the origin (0, 0) to a point (x, y) is given by the distance formula.

Distance and Midpoint Formulas

For two points A(x •, y •) and B(x ,, y ,):

- Distance: d = " [(x ," x •)² + (y ," y •)²]
- Midpoint: M((x •+x ,)/2, (y •+y ,)/2)

Slope of a Line and Linear Equations

The slope (or gradient) of the line passing through two points is m = (y ," y •)/(x ," x •). Using the point slope form, the equation of a line can be written as:

y" y • = m(x" x •)

In slope intercept form, it becomes y = mx + b, where b is the y intercept.

Equations of Conic Sections

Conic sections are the curves obtained by intersecting a plane with a right circular cone. Their standard equations in the Cartesian plane are:

- Circle: (x" h)² + (y" k)² = r²
- Parabola (vertical axis): y = ax² + bx + c or (x" h)² = 4p(y" k)
- Ellipse: (x" h)²/a² + (y" k)²/b² = 1
- Hyperbola: (x" h)²/a² " (y" k)²/b² = 1

These equations allow us to analyze properties such as foci, directrices, axes, and asymptotes.

Problem Set 1: Distance, Midpoint, and Slope

Problem 1: Distance Between Two Points

Find the distance between points A(3, " 4) and B(" 1, 2).

Solution 1

Using the distance formula:

9G, Ò ,!# ’ !" 2 Ò !#@

9G’ Ò " !" ,!#B’ Ò `

d = " [(" 4)² + 6²] = " [16 + 36] = " 52 = 2" 13 "H 7.21

Problem 2: Midpoint of a Line Segment

Find the midpoint of the segment connecting P(" 2, 5) and Q(4, " 3).

Solution 2

Midpoint formula:

Mx = (" 2 + 4)/2 = 1

My = (5 + (" 3))/2 = 1

Midpoint M = (1, 1).

Problem 3: Slope of a Line Through Two Points

Determine the slope of the line passing through R(0, 3) and S(5, 11).

Solution 3

m = (11" 3)/(5" 0) = 8/5 = 1.6.

Problem 4: Equation of a Line Using Point Slope Form

Find the equation of the line that passes through T(2, " 1) and has a slope of " 3.

Solution 4

Using point slope: y" (" 1) = (" 3)(x" 2)

y + 1 = " 3x + 6

y = " 3x + 5.

Problem 5: Parallel and Perpendicular Lines

Given the line 3x " 4y + 7 = 0, find the equations of:

1. The line parallel to it that passes through (1, 2).
2. The line perpendicular to it that passes through (" 3, 5).

Solution 5

First, rewrite the given line in slope intercept form.

3x " 4y + 7 = 0 !' 4y = 3x + 7 !' y = (3/4)x + 7/4.

Thus, the slope of the given line is m = 3/4.

1. Parallel line: same slope m = 3/4. Use point slope with (1, 2):

y" 2 = (3/4)(x" 1)

y = (3/4)x + 2 " 3/4 = (3/4)x + 5/4.

1. Perpendicular line: slope m" = " 4/3. Use point (" 3, 5):

y" 5 = (" 4/3)(x+3)

y = (" 4/3)x + (" 4) + 5 = (" 4/3)x + 1.

Problem Set 2: Equations of Conic Sections

Problem 6: Equation of a Circle

The circle has center at (" 2, 5) and passes through the point (3, 1). Write its equation.

Solution 6

Radius r = distance between center and given point.

9G, Ò 2!",!#"' Ò P

9G' Ò !#R Ò !#@

r² = 5² + (" 4)² = 25 + 16 = 41.

Equation: (x+2)² + (y" 5)² = 41.

Problem 7: Parabola Vertex Form

Find the vertex form of the parabola whose standard equation is y = 2x² " 12x + 18.

Solution 7

Complete the square.

y = 2(x² " 6x) + 18

= 2[(x" 3)² " 9] + 18

= 2(x" 3)² " 18 + 18

= 2(x" 3)².

Thus, vertex form: y = 2(x" 3)², vertex at (3, 0).

Problem 8: Ellipse with Given Foci and Major Axis

An ellipse has foci at (" 3, 0) and (3, 0), and the length of the major axis is 10. Determine the standard equation of

the ellipse.

Solution 8

Center is at the midpoint of the foci: (0, 0).

Distance between foci = 6 !’ 2c = 6 !’ c = 3.

Major axis length = 2a = 10 !’ a = 5.

For ellipse, $c^2 = a^2$ " b^2 !’ 9 = 25 " b^2 !’ b^2 = 16 !’ b = 4.

Standard equation: $x^2/25 + y^2/16 = 1$.

Problem 9: Hyperbola with Asymptotes

The hyperbola has asymptotes y = 2x and y = " 2x, and passes through the point (4, 6). Find its equation.

Solution 9

Since asymptotes cross at the center, center is at the origin (0,0).

The slopes of the asymptotes are ±b/a = ±2. Thus, b/a = 2.

Equation of hyperbola: x^2/a^2 " $y^2/b^2 = 1$.

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